Nonlocal electron kinetics in a weakly ionized plasma

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(Received 5 February 1998)

Electron dynamics in a time dependent inhomogeneous electric field in a weakly ionized plasma with elastic electron-neutral collisions is analyzed. We consider the most general ordering when the electron mean free path v_{Te}/ν_e is arbitrary with respect to the characteristic length scale k^{-1} of the electric field, and frequency ω of the electric field is arbitrary with respect to the electron collisional frequency ν_e; $\omega \sim \nu_e \sim kv_t$. In this case the standard two-term approximation is not valid and higher order spherical harmonics in the perturbed electron distribution function should be taken into account. This results in an infinite hierarchy of coupled equations for angular harmonics that can be solved in the form of the infinite continued fraction. This method is easily generalized for a wide class of scattering cross sections with angular dependencies. The developed approach uniformly describes both local (strongly collisional) and nonlocal regimes. As an example, a closed form of the perturbed electron distribution function is found for the argon gas with nonmonotic dependence of the collisional cross section as function of energy (Ramsauer effect). The conductivity and surface impedance of a semi-infinite plasma are calculated in different collisionality regimes, and anomalous penetration of the electric field into such plasma is analyzed. The nonmonotonous behavior of the amplitude of the external electric field inside of a plasma has been recovered for the nonlocal case ($\zeta > 1$). [S1063-651X(98)06407-1]

PACS number(s): 51.50.+v, 51.10.+y, 52.20.−j, 52.50.−b

I. INTRODUCTION

Electron transport in an external electric field in a weakly ionized plasma plays a fundamental role in gas discharge physics and its applications, in particular, for a variety of plasma sources used in science and technology. In the present paper we consider the electron conductivity of a low temperature plasma in a time dependent nonuniform electric field when the main mechanism of electron scattering is elastic collisions with neutrals and analyze nonclassical (anomalous) penetration of the electric field into in a situation when effects of electron thermal motion are important.

We are interested in a situation when the electron mean free path is not small compared to the characteristic time scale of the external electric field inhomogeneity, and the electron collisional frequency is not necessary large compared to the characteristic frequency of the electric field oscillations. For such conditions effects of the thermal electron motion become important so that electron conductivity becomes a nonlocal operator both in space and in time [1,2]. This modifies the mechanism of the electric field penetration into a plasma that is no longer local but rather becomes nonlocal. It is usually referred to as an anomalous skin effect [3,4]. These effects have been observed experimentally in gas discharges [5,6]. They are becoming increasingly important as gas discharges for plasma processing and lighting move toward the lower pressure regimes. In this paper we develop an approach that allows one to uniformly describe both strongly collisional and collisionless regimes. Such an approach is especially important for the description of the electron transport in inert gases where the differential cross section of the electron-neutral atom interaction exhibits nonmonotonic behavior with the electron energy (the Ramsauer effect) so that electrons of different energies could be in different collisionality regimes.

The traditional procedure to solving the Boltzmann kinetic equation for electrons is to expand the electron distribution function in a series of spherical functions and then truncate this representation retaining only a certain number of terms [7]. This series expansion is equivalent to the expansion of the distribution function in the parameter $kv/(i\omega - \nu)$ where k is the wave number, ω is the frequency of the external electric field, ν is the collision frequency, and v is the electron velocity. Often, the so-called two-term approximation [3,8,9] is implemented to solve the Boltzmann equation. The applicability of the two-term expansion to the problem of the electron conductivity is limited to the case when the parameter $kv_T/(i\omega - \nu)$ is sufficiently small, i.e., the electron motion is strongly collisional, $\nu > \omega$, $\nu > kv_T$, or the oscillation frequency is large, $\omega > \nu, \omega > kv_T$. In this approximation, the perturbed distribution function and, respectively, the electron conductivity do not depend on the wave vector k. Thus, the electron conductivity is a local quantity and does not take into account the thermal electron motion. It is worth noting also that the two-term approximation does not describe higher order moments such as electron heat flux which is important for the problem of anomalous heating [10,11]. The two-term approximation can be further improved by including a few more terms in the spherical harmonic expansion [12]. We develop an approach that includes complete infinite hierarchy of spherical harmonics and allows one to calculate the perturbed distribution function for arbitrary values of the parameter $kv/(i\omega - \nu)$. In present paper we concentrate on the nonlocal effects in the perturbed distribution function due to the external electric field and neglect effects of inhomogeneity of the equilibrium electron density due to the equilibrium ambipolar potential. Thus we neglect effects of energy relaxation and associated nonlocal effects due to particle trapping in the ambipolar potential of a bounded plasma column [3,9]. The latter effects can be

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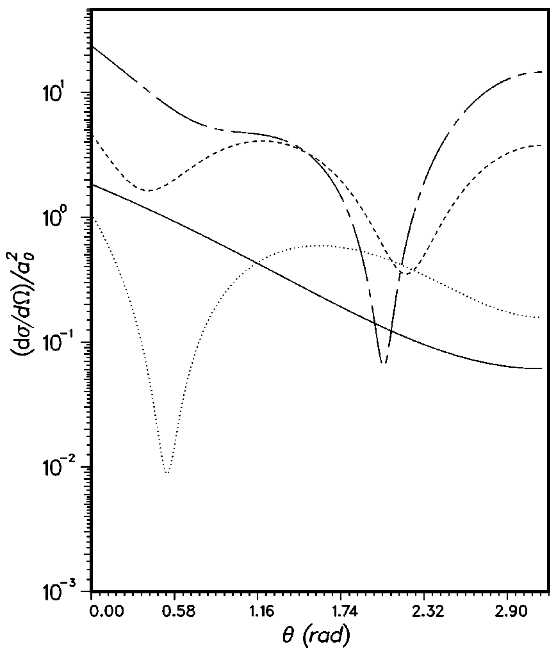

FIG. 1. The differential cross section of scattering of an electron on argon atom vs the scattering angle for different electron energies. The solid, dotted, dashed, and dashed-dotted lines represent the electron energy of 0.1, 1, 5, and 10 eV, respectively.

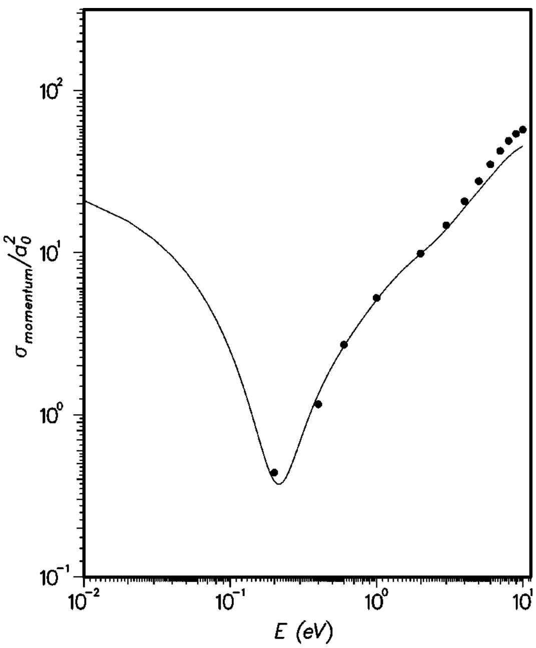

FIG. 2. The momentum transfer scattering cross section of an electron on argon versus the electron energy. ●: Naskanishi *et al.* (Ref. [23]); —: present results.

described within the two-term approximation and are not considered in this paper.

Nonlocal electron conductivity and the associated anomalous skin effect have also been investigated [1,3,13,14] by solving the Boltzmann kinetic equation with simplified BGK type collisional operator in the form

$$C(f)=-\nu(v)f, \tag{1}$$

where $\nu(v)$ is the effective collisional frequency. Using Eq. (1) one can solve the linearized kinetic equation for perturbations without making the expansion in spherical harmonics. Such an approximation may be used for electrons of very low energies where the differential cross section of the electron-atom interaction does not depend on the electron energy and is isotropic in space. At higher energies of electrons the cross section of the electron-neutral interaction becomes velocity dependent and anisotropic in space. This is usually the most typical situation for electron scattering in inert gases with the Ramsauer effect. In this paper we consider the realistic case when the differential cross section is anisotropic and approximation (1) is not valid. As an example we use argon gas where the Ramsauer effect manifests itself in a sharp decrease in the magnitude of the transport cross section as the energy of electrons increases. In the region of the minimum energy, the spatial motion of electrons may become important even if it can be neglected for other energy ranges.

II. SOLUTION OF THE ELECTRON KINETIC EQUATION IN THE INHOMOGENEOUS AND TIME DEPENDENT ELECTRIC FIELD

The Boltzmann kinetic equation for the electron distribution function in a weakly ionized plasma with the external time-varying and inhomogeneous electric field is

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}-\frac{e\mathbf{E}(\mathbf{r},t)}{m}\cdot\frac{\partial f}{\partial \mathbf{v}}=C(f). \tag{2}$$

Here C is the collisional integral of interaction between electrons and atoms given by [15]

$$C(f)=\int v_{\mathrm{rel}}(f'f_1'-ff_1)\frac{d\sigma}{d\Omega}d^3p_1 d\Omega, \tag{3}$$

where v_{rel} is the relative velocity between electrons and atoms, f, f_1 and f', f_1' are the electron and neutral atom distribution functions before and after the collision, $d\sigma/d\Omega$ is the differential cross section of electron-neutral collisions. In the present paper we neglect the electron-electron and electron-ion interactions, since the neutral atom density is considerably higher then that of the electrons and ions.

We neglect the effects of energy transfer between electrons and atoms because of the large mass difference between the two species. Thus atoms are motionless, and their distribution function is given by

$$f_1'=f_1=N\delta(p_{1x})\delta(p_{1y})\delta(p_{1z}), \tag{4}$$

where N is the neutral atom density. Substituting the expression (4) into (3) we get

$$C=N\int [f(t,\mathbf{r}',\mathbf{v}')-f(t,\mathbf{r},\mathbf{v})]v\frac{d\sigma}{d\Omega}d\Omega. \tag{5}$$

As mentioned above we neglect the effects of spatial distribution of the ambipolar potential, so that the electron equilibrium function $F_0(v)$ can be taken in the Maxwellian form with a uniform density. Under the influence of the external electric field, the electron distribution function departs from

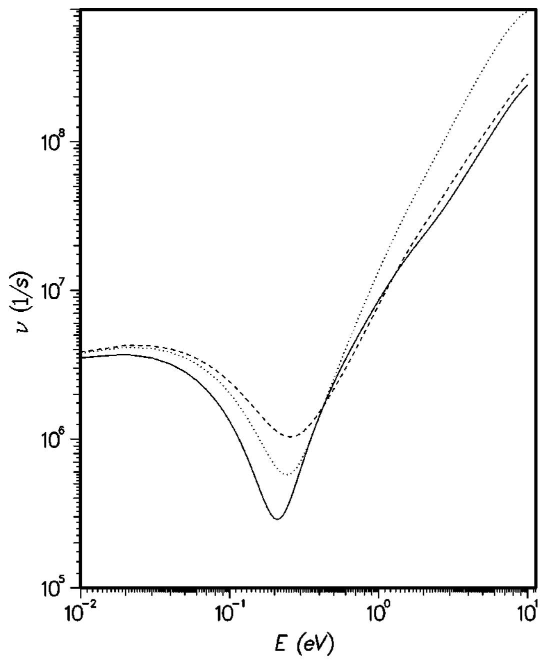

FIG. 3. The effective collision frequencies ν_1, ν_2, and ν_∞ versus the electron energy. The neutral atom density is 10^{15} cm^{-3}. The solid line represents ν_1, the dotted ν_2, and the dashed ν_∞.

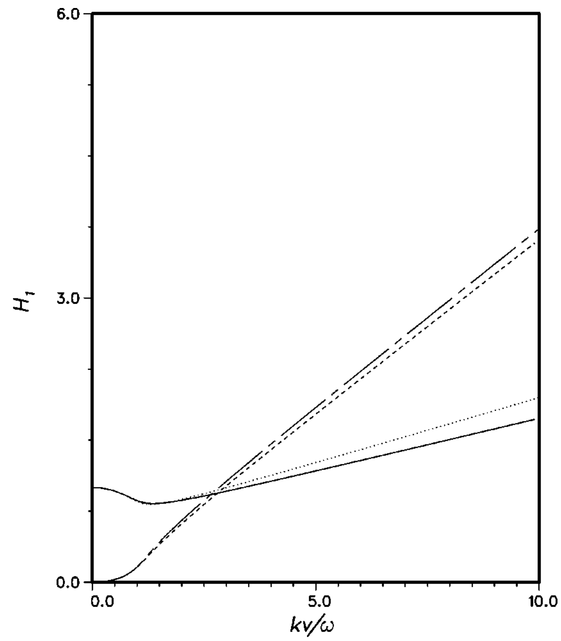

FIG. 4. The complete continued fraction H_1 computed numerically and the approximation (A7) as the function of the kv/ω parameter for $\nu_1/\omega=0.1$. The solid line: the real part of the complete continued fraction H_1; the dotted line: the real part of the approximation (A7); the dashed line: the imaginary part of the complete continued fraction H_1; the dashed-dotted line: the imaginary part of the approximation (A7).

the equilibrium so that the total electron distribution function can be represented in the form [7]

$$f(\mathbf{v},\mathbf{r},t)=F_0(v)+\sum_{l,m} f_{l,m}(v)Y_{l,m}(\theta,\phi)e^{i(kx-\omega t)}, \quad (6)$$

where $v=|\mathbf{v}|$, $f_{l,m}(v)$ is a function of the absolute value of electron velocity, $Y_{l,m}(\theta,\phi)$ is the spherical harmonic function, and F_0 is the Maxwellian distribution function. We assume that the electric field is in the z direction, $\mathbf{E}=E\hat{\mathbf{z}}$, and its variation is in the x direction, $\mathbf{k}=k\hat{\mathbf{x}}$.

Substituting Eq. (6) into Eqs. (5) and (2), and expanding the electric field in Fourier series, one gets in the linear approximation

$$-i\omega\sum_{l,m} f_{l,m}(v)Y_{l,m}(\theta,\phi)+ikv\sin\theta\cos\phi\sum_{l,m} f_{l,m}(v)Y_{l,m}(\theta,\phi)-\frac{eE(\omega,k)}{m}\cos\theta\frac{\partial F_0}{\partial v}=-\sum_{l,m}\nu_l f_{l,m}(v)Y_{l,m}(\theta,\phi), \quad (7)$$

where ν_l is the lth order collision frequency defined by [7]

$$\nu_l(v)=Nv\int[1-P_l(\cos\theta)]\frac{d\sigma(v,\theta)}{d\Omega}d\Omega, \quad (8)$$

where $P_l(\cos\theta)$ are the Legendre polynomials. If the differential cross section does not depend upon the poloidal scattering angle θ, then all ν_l are equal, except for ν_0. The characteristic frequency ν_0 describes the energy relaxation that is neglected here due to large mass difference between the electron and neutral atoms. If $\nu_l=$const for all l then there is no need to expand the electron distribution function in the series of spherical functions and one can solve the Boltzmann kinetic equation with the collision term given by Eq. (1).

If the differential cross section is a function of θ, Eq. (7) leads to coupled equations for different spherical harmonics. Multiplying Eq. (7) by $Y^*_{l,m}(\theta,\phi)$ and integrating over the solid angle we obtain

$$-i\omega f_{l,m}+\frac{1}{2}ikv\Gamma_{l,m}-\sqrt{\frac{4\pi}{3}}\frac{eE(\omega,k)}{m}\delta_{l,1}\delta_{m,0}\frac{\partial F_0}{\partial v}=-\nu_l f_{l,m}, \quad (9)$$

where

$$\Gamma_{l,m}=\sqrt{\frac{(l-m+1)(l-m+2)}{(2l+1)(2l+3)}}f_{l+1,m-1}+\sqrt{\frac{(l-m)(l-m-1)}{(2l+1)(2l-1)}}f_{l-1,m+1}-\sqrt{\frac{(l+m-1)(l+m)}{(2l-1)(2l+1)}}f_{l-1,m-1}$$
$$-\sqrt{\frac{(l+m+2)(l+m+1)}{(2l+1)(2l+3)}}f_{l+1,m+1}. \tag{10}$$

The standard two-term approach consists in retaining in Eq. (9) only the first term, f_{10}, and neglecting all the higher angular harmonics. This is justifiable in a strongly collisional limit, but in weakly collisional regimes effects of the higher angular harmonics of the distribution function become important. These effects can be accounted for by a direct solution of the infinite system (9) for $f_{1,0}(\omega,k,v)$ in the form

$$f_{1,0}(\omega,k,v)=-\sqrt{\frac{4\pi}{3}}\frac{eE(\omega,k)}{m}\frac{\partial F_0}{\partial v}\frac{1}{[i\omega-\nu_1(v)]H_1(v,k,\omega)}. \tag{11}$$

The effects of the higher-order spherical harmonics are included in the continued fraction

$$H_l(v,k,\omega)=1+C_{l+1}/(1+C_{l+2}/1+C_{l+3}/\cdots), \tag{12}$$

with coefficients

$$C_l=\frac{(l^2-1)k^2v^2}{(4l^2-1)(i\omega-\nu_l)(i\omega-\nu_{l-1})}. \tag{13}$$

The similar method of incorporating of the higher order spherical harmonics was used in [16,17] for the problem of electron-ion collisions.

To find an expression for the perturbed electron distribution function, one has to calculate the continued fraction $H_1(v,k,\omega)$, which in turn requires the knowledge of the lth order collision frequencies ν_l. If the electron-neutral atom interaction is of the polarization type, which is assumed in the present paper, then the collision frequency ν_l rapidly converges to a constant value

$$\nu_\infty(v)=\lim_{l\to\infty}\nu_l(v). \tag{14}$$

In this case the continued fraction $H_1(v,k,\omega)$ can be approximated by (see Appendix A)

$$H_1(\omega,kv,\nu)=1+\frac{2}{5}\frac{k^2v^2}{(i\omega-\nu_2(v))(i\omega-\nu_1(v))(\sqrt{1+k^2v^2/(i\omega-\nu_\infty(v))^2}+1)}, \tag{15}$$

where

$$\nu_1(v)=Nv\int(1-\cos\theta)\frac{d\sigma(v,\theta)}{d\Omega}d\Omega, \tag{16}$$

$$\nu_2(v)=\frac{3}{4}Nv\int(1-\cos 2\theta)\frac{d\sigma(v,\theta)}{d\Omega}d\Omega. \tag{17}$$

Substituting Eq. (15) into Eq. (11) we obtain the expression for the electron perturbed distribution function in the form

$$f_{1,0}(k,\omega,v,\theta)=-\frac{eE(\omega,k)}{m}\frac{\partial F_0}{\partial v}\frac{\cos\theta}{i\omega-\nu_1(v)}\left(1+\frac{2}{5}\frac{k^2v^2}{[i\omega-\nu_2(v)](i\omega-\nu_1(v))(\sqrt{1+k^2v^2/[i\omega-\nu_\infty(v)]^2}+1)}\right)^{-1}. \tag{18}$$

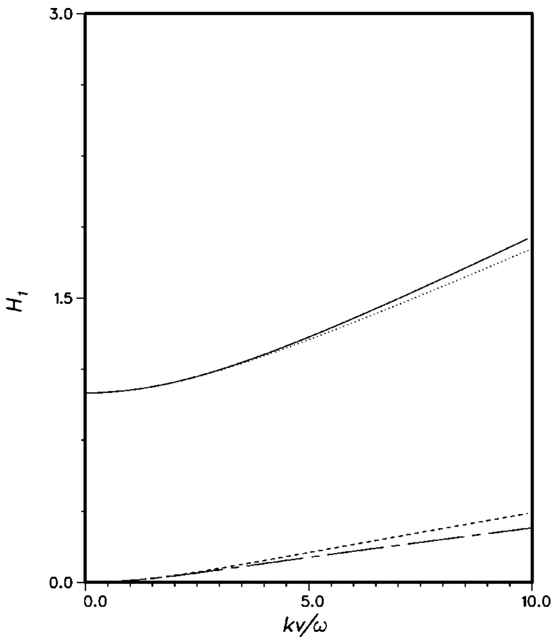

FIG. 5. The complete continued fraction H_1 computed numerically and the approximation (A7) as the function of the kv/ω parameter for $\nu_1/\omega=5$. The solid line: the real part of the complete continued fraction H_1; the dotted line: the real part of the approximation (A7); the dashed line: the imaginary part of the complete continued fraction H_1; the dashed-dotted line: the imaginary part of the approximation (A7).

In the limit, $k=0$ we obtain the local electron distribution function from Eq. (18), investigated in Ref. [8]. We have parametrized the perturbed distribution function $f_{1,0}$ by three collisional frequencies ν_1, ν_2, and ν_∞. As shown in the next section, this approximation satisfactorily describes the particular case of electron scattering in the argon gas. The parametrization (15) can be easily generalized for the arbitrary dependence of the collisional cross section $d\sigma(v,\theta)/d\Omega$, provided that the condition (14) is satisfied. It is worth noting that both analytical representation and results of computer modeling and/or experimental data can be used for $d\sigma(v,\theta)/d\Omega$ to determine the parameters ν_1,ν_2, and ν_∞ required for Eq. (18).

III. EFFECTIVE COLLISIONAL FREQUENCIES AND RAMSAUER EFFECT IN ARGON

To calculate the perturbed distribution function (18) one has to specify the form of the collisional integral and then calculate ν_l. As noted before, in this paper we consider the elastic electron-atom polarization interaction as the main channel of electron scattering. For the incident electron energy in the range between 0 and 10 eV, the differential cross section of the electron-atom interaction can be represented in the form (B12) (see Appendix B). The structure of the energy expansion of phase shifts for the interaction between electrons and the neutral polarizable system (atoms) follows from the so-called modified (atomic) effective range theory [18].

The differential cross section (B12) is a function of the electron velocity v and the scattering angle θ. For some gases (e.g., argon), the scattering length L is negative and the transport cross section $\sigma_{\rm tr}$, defined as

$$\frac{\sigma_{\rm tr}^{\rm arg}}{4\pi}=\frac{1}{4\pi}\int(1-\cos\theta)d\sigma$$

$$=\hat{A}\hat{A}^*+\frac{\hat{B}\hat{B}^*}{3}+\frac{\hat{C}\hat{C}^*}{5}$$

$$-\frac{2}{5}\frac{\pi\beta k}{a_0}\{\hat{A}+\hat{A}^*\}+\frac{18\pi\beta k}{105a_0}\{\hat{B}+\hat{B}^*\}$$

$$-\frac{2\pi\beta k}{105a_0}\{\hat{C}+\hat{C}^*\}-\frac{1}{3}\{\hat{A}\hat{B}^*+\hat{A}^*\hat{B}\}$$

$$-\frac{2}{15}\{\hat{B}\hat{C}^*+\hat{B}^*\hat{C}\}+\frac{1}{6}\frac{\pi^2\beta^2k^2}{a_0^2}\qquad(19)$$

has a minimum at $v\simeq v_c$ [19] where

$$v_c=-\frac{12\hbar La_0}{5\pi m\beta}.\qquad(20)$$

The sharp decrease in the transport cross section of electron atom interaction is known as the Ramsauer effect. As an example we shall use argon gas for the present calculations. The atomic polarizability and the scattering length of the argon atom are [18,20] $\beta=11.1a_0^3$, $L=-1.7a_0$, where a_0 is the Bohr radius, and the coefficients β_l,γ_l,j_l (given in Appendix B) are chosen to fit the experimental data [21] for the phase shifts in the energy range from 0 to 10 eV. The angular dependence of the differential cross section for different energies given by Eq. (B12) is shown in Fig. 1. For the energy range between 0 and 10 eV, the expression (B12) gives a very good fit to the differential cross section of the electron-argon atom interaction that was obtained experimentally [22,24,25] and theoretically [22,24,25].

The transport cross section (19) of electron scattering in argon as a function of the incident electron energy is shown in Fig. 2. Using the definition of the lth order collision frequency (8) and (B12) one obtains

$$\frac{\nu_l(v)}{Nv}=4\pi\hat{A}\hat{A}^*(1-\delta_{l,0})+\frac{\pi^3\beta^2k^2}{2a_0^2}\left(1-\delta_{l,0}+\frac{1}{3}\delta_{l.1}\right)-\frac{4\pi^2\beta k}{a_0}\{\hat{A}+\hat{A}^*\}\left(\frac{1}{3}+\frac{1}{(4l^2-1)(2l+3)}\right)-\frac{4\pi}{3}\{\hat{A}\hat{B}^*+\hat{A}^*\hat{B}\}\delta_{l.1}$$
$$+\frac{2\pi^2\beta k}{a_0}\{\hat{B}+\hat{B}^*\}\left(\frac{2}{15}-\frac{2(l+1)}{(2l+1)^2(2l+5)(2l+3)}-\frac{2l}{(2l+1)^2(2l-3)(2l-1)}\right)+4\pi\hat{B}\hat{B}^*\left(\frac{1}{3}-\frac{1}{3}\delta_{l,0}-\frac{2}{15}\delta_{l,2}\right)$$
$$+2\pi\hat{C}\hat{C}^*\left(\frac{2}{5}-\frac{2}{5}\delta_{l,0}-\frac{4}{35}\delta_{l,2}-\frac{4}{35}\delta_{l,4}\right)-\frac{4\pi}{2}\{\hat{A}\hat{C}^*+\hat{A}^*\hat{C}\}\delta_{l,2}-2\pi\{\hat{B}\hat{C}^*+\hat{B}^*\hat{C}\}\left(\frac{4}{15}\delta_{l.1}+\frac{6}{35}\delta_{l.3}\right)$$
$$-\frac{\pi^2\beta k}{a_0}\{\hat{C}+\hat{C}^*\}\left(-\frac{4}{105}-\frac{2}{(2l+3)(4l^2-1)}+\frac{6(l+1)^2}{(2l+1)^2(2l+3)^2(2l-1)}\right)-\frac{\pi^2\beta k}{a_0}\{\hat{C}+\hat{C}^*\}$$
$$\times\left(\frac{6(l+1)(l+2)}{(2l+1)(2l+3)^2(2l+5)(2l+7)}+\frac{6l(l-1)}{(2l+1)(2l-3)(2l-1)^2(2l-5)}+\frac{6l^2}{(2l+1)^2(2l-1)^2(2l+3)}\right). \quad (21)$$

Figure 3 shows the effective collision frequencies ν_1, ν_2, and ν_∞ versus the electron energy. One can see the nonmonotonic behavior of the effective frequencies with the electron energy, which is typical for the Ramsauer effect. With an explicit form for the effective collision frequencies (21) we can readily calculate the continued fraction H_1 and subsequently the perturbed electron distribution function. The approximation (15) reasonably well reproduces the exact continued fraction (12) in the wide range of collisionality regimes. The comparison of the full continued fraction (12) computed numerically with the approximation (15) is given in Figs. 4 and 5 for different values of the parameter ν_1/ω.

IV. NONLOCAL PLASMA CONDUCTIVITY

The Fourier component of the plasma conductivity is defined from the relation

$$j(\omega,k)=-e\int v_z f_{1,0}d^3v=\sigma(\omega,k)E(\omega,k). \quad (22)$$

Using Eq. (18) one obtains

$$\sigma(\omega,k)=-\frac{4\pi ne^2}{3T}\left(\frac{m}{2\pi T}\right)^{3/2}\int_0^\infty\frac{v^4\exp(-mv^2/2T)}{i\omega-\nu_1(v)}\left(1+\frac{2}{5}\frac{k^2v^2}{[i\omega-\nu_2(v)][i\omega-\nu_1(v)](\sqrt{1+k^2v^2/(i\omega-\nu_\infty(v))^2}+1)}\right)^{-1}dv. \quad (23)$$

The effective frequencies defined in this paper reproduce well theoretical and experimental data in the energy region from 0 to 10 eV. Integration in the expression (23) is done over the entire velocity region from 0 to ∞. Since the main contribution to the integral in Eq. (23) come from velocities that are close to the thermal, the error introduced by electrons with energies higher than 10 eV is exponentially small for plasmas with electron temperature $T_e<10$ eV.

The electron plasma conductivity σ given by Eq. (23) is a complicated function of ω and k. Converted back to a configuration space z and t, $\sigma(z,t)$ becomes a nonlocal integro-differential operator in space and time. It means that the conductivity is a function of the rf field throughout the entire skin layer and as will be shown in the next section it causes the nonmonotonic behavior of the electric field inside of a plasma.

For the case when $\nu=0$, the plasma conductivity can be easily calculated from the collisionless kinetic equation giving

$$\sigma_0(\omega,k)=-\frac{i\omega_{pe}^2}{4\pi kv_T}Z\left(\frac{\omega}{kv_T}\right), \quad (24)$$

where $Z(x)$ is the plasma dispersion function

$$Z(x)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\exp(-t^2)}{t-x}dt.$$

In Fig. 6 we compare the exact analytical result (24) obtained in the collisionless regime with our approximation (23) in the limit $\nu\to0$. This comparison demonstrates that Eq. (23) accurately describes the plasma response for a wide range of the collisionality regimes and a wide range of values of the parameter ω/kv_T.

The second term in brackets in expression (23) describes nonlocal effects due to a spatial dependence of the conductivity operator σ. One can introduce the following criterion when the nonlocal effects are important:

$$\zeta\equiv\left|\frac{k^2v_T^2}{(i\omega-\nu_2)(i\omega-\nu_1)\{[1+k^2v_T^2/(i\omega-\nu_\infty)^2]^{1/2}+1\}}\right|>1. \quad (25)$$

Qualitatively we can assume that $\nu_1=\nu_2=\nu_\infty$. Then expression (25) can be rewritten as

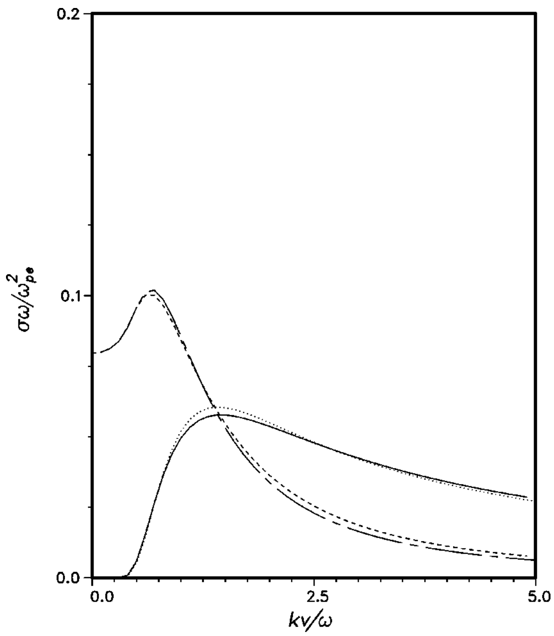

FIG. 6. The collisionless plasma conductivity (24) and plasma conductivity (23) calculated in the limit $\nu\to 0$. The solid line: the real part of the collisionless plasma conductivity (24); the dotted line: the real part of plasma coductivity (23) calculated in the limit $\nu\to 0$; the dashed line: the imaginary part of the collisionless plasma conductivity; the dashed-dotted line: the imaginary part of plasma conductivity (23) calculated in the limit $\nu\to 0$.

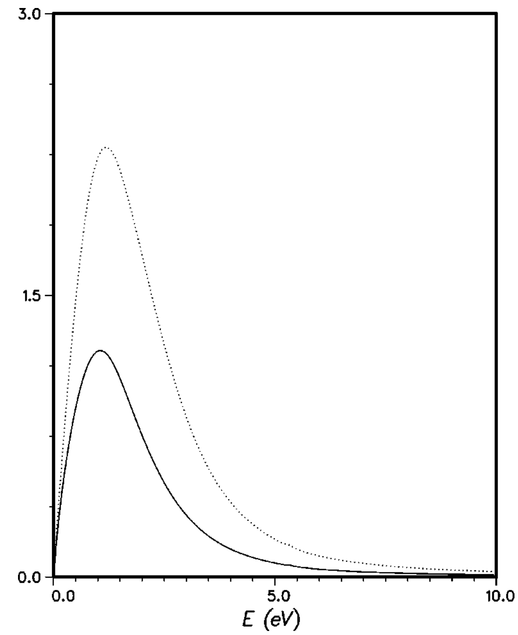

FIG. 7. The nonlocality parameters ζ and Λ vs temperature for the neutal atom density $N=10^{15}$ cm^{-3}, electron density $n_e=10^{12}$ cm^{-3}, and the frequency of the external electric field $\omega=6.78$ MHz. The solid line represents ζ and the dotted line represents Λ.

$$\frac{k^2v_T^2}{\omega^2+\nu_1^2}>[1+k^2v_T^2/(\omega^2+\nu_1^2)]^{1/2}+1. \tag{26}$$

Introducing a parameter η

$$\eta=\frac{k^2v_T^2}{\omega^2+\nu_1^2} \tag{27}$$

one obtains from Eq. (26)

$$\eta>(1+\eta)^{1/2}+1. \tag{28}$$

Thus the nonlocal effects are important for

$$\eta>1 \tag{29}$$

or

$$\eta=\frac{k^2v_T^2}{\omega^2+\nu_1^2}>1. \tag{30}$$

The characteristic wave vector k can be approximated by the inverse skin depth in the local case

$$k=\delta^{-1}=\mathrm{Re}(4\pi i\omega\sigma/c^2)^{1/2}, \tag{31}$$

where σ is the local conductivity. By using the classical local expression for the electron conductivity

$$\sigma=\frac{\omega_p^2}{4\pi(\nu-i\omega)}, \tag{32}$$

one obtains from Eqs. (30) and (31) well known nonlocality parameter Λ introduced in ([1,13])

$$\Lambda=\left(\frac{\omega_p v_T}{c}\right)^2\frac{\omega}{(\omega^2+\nu^2)^{3/2}}. \tag{33}$$

The expression (32) is valid, however, only in the limit when the effective collisional frequency ν does not depend on the electron velocity. It has been pointed out in Ref. [8] that for the velocity dependent collisional frequency $\nu(v)$ the expression (32) does not provide an adequate description of the electron conductivity σ. In the latter case, more general expression

$$\sigma=-\frac{4\pi ne^2}{3T}\left(\frac{m}{2\pi T}\right)^{3/2}\int_0^\infty\frac{v^4\exp(-mv^2/2T)dv}{i\omega-\nu_1(v)}. \tag{34}$$

has to be used for evaluation of the local conductivity. Respectively, the more general expression (34) has to used in Eq. (31) to evaluate the characteristic wavelength k. The parameters ζ (25) with k calculated by using the local conductivity σ from (34) and Λ (33) are plotted in Fig. 7 as functions of the electron energy for the same values of the electron and neutral atoms density and frequency of the external field. As one can see there is a substantial difference between these two parameters. The parameter Λ considerably overestimates the nonlocality region for this particular case of argon gas.

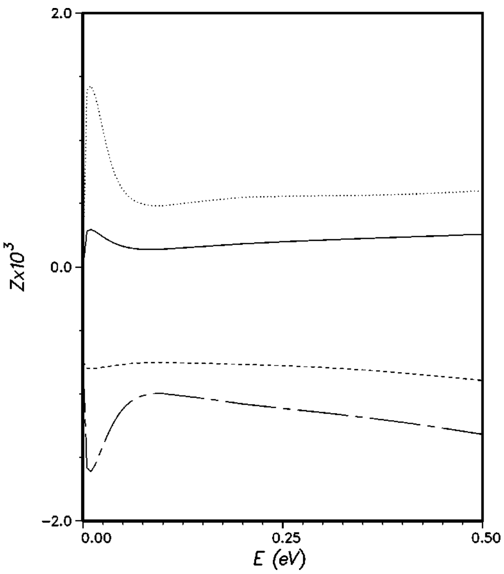

FIG. 8. The surface impedance vs the electron energy. The solid line: the real part of the system's surface impedance for the neutral atom and electron densities of $N_{\rm at}=10^{15}$ cm^{-3}, $n_e=10^{12}$ cm^{-3}; the dashed line: the imaginary part of system's surface impedance for the neutral atom and electron densities of $N_{\rm at}=10^{15}$ cm^{-3}, $n_e=10^{12}$ cm^{-3}. The dotted line: the real part of system's surface impedance for the neutral atom and electron densities of $N_{\rm at}=10^{16}$ cm^{-3}, $n_e=10^{12}$ cm^{-3}; dashed-dotted line: the imaginary part of system's surface impedance for the neutral atom and electron densities of $N_{\rm at}=10^{16}$ cm^{-3}, $n_e=10^{12}$ cm^{-3}. The electric field frequency is $\omega=6.28$ MHz.

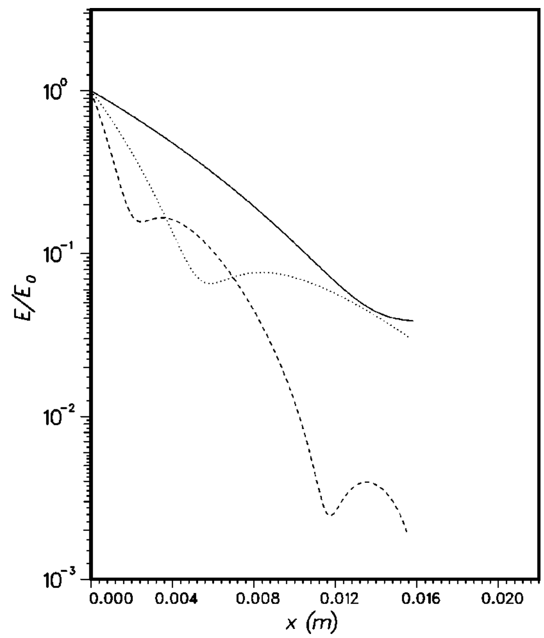

FIG. 9. The amplitude of the electric field E as a function of the distance x for different values of parameter ζ. The solid line: $\xi=0.9$ ($N_{\rm at}=10^{15}$ cm^{-3}, $n_e=10^{12}$ cm^{-3}, $\omega=6.78$ MHz, $T_e=0.5$ eV); the dotted line: $\xi=5.2$ ($N_{\rm at}=10^{15}$ cm^{-3}, $n_e=10^{13}$ cm^{-3}, $\omega=6.78$ MHz, $T_e=1.0$ eV); and the dashed line: $\xi=30.3$ ($N_{\rm at}=10^{15}$ cm^{-3}, $n_e=10^{14}$ cm^{-3}, $\omega=3.89$ MHz, $T_e=0.6$ eV).

V. SURFACE IMPEDANCE OF THE SEMI-INFINITE WEAKLY IONIZED PLASMA

In this section we analyze the electric field penetration into a semi-infinite plasma $x>0$ and calculate the surface impedance when the spatial motion of electrons becomes important. Assuming the specular reflection of the electrons at the boundary $x=0$, the profile of the electric field in a semi-infinite plasma is given by [26]

$$E_z(x)=-i\frac{\omega}{\pi c}B_y(+0)\int_{-\infty}^{\infty}\frac{\exp[ikx]dk}{k^2-4\pi i\omega\sigma(\omega,|k|)/c^2}. \tag{35}$$

The surface impedance Υ is defined as

$$\Upsilon=\frac{E_z(0)}{B_y(0)}=-i\frac{\omega}{\pi c}\int_{-\infty}^{\infty}\frac{dk}{k^2-i4\pi\omega\sigma(|k|,\omega)/c^2}, \tag{36}$$

where $\sigma(|k|,\omega,T)$ is the Fourier component of plasma conductivity.

The nonlocality parameter ζ shows that in both limits of low and high frequency as well as low and high temperature cases the penetration of the electromagnetic waves into a plasma can be described as a classical skin effect (exponential decay of an electric field inside of a plasma) and the surface impedance can be calculated by integrating the expression (36) with the conductivity given by Eq. (34), which finally yields [27]

$$Z=(1-i)\sqrt{\frac{\omega}{8\pi\sigma}}. \tag{37}$$

For the nonlocal case ($\zeta>1$) however, the formula (37) is not applicable and one has to solve the problem of propagation of the electromagnetic wave, when the current at a given point is determined by the field distribution within the electron free path distance. The electric field and impedance numerically calculated from Eqs. (36) and (35) are shown in Figs. 8 and 9. In the local case $\zeta\leqslant 1$ the electric field amplitude monotonically decays with the distance (approximately as a damped exponent). For the nonlocal case $\zeta>1$, on the contrary, the nonmonotonic field decay occurs. For the gas pressures of the order of 10^{-2} Torr the system's surface impedance becomes a nonmonotonic function of temperature, which is the result of the Ramsauer effect.

VI. CONCLUSIONS

In the present paper, we have considered nonlocal electron kinetics in a weakly ionized plasma subject of the time and space dependent electric field when the main channel of electron scattering is the electron-neutral atom interaction. Since the differential cross section of electron scattering depends on the poloidal angle, the collision integral $C(f)$ cannot be presented in the local form $C(f)=-\nu(v)f(\mathbf{r},\mathbf{v},t)$. In this case the perturbed electron distribution function is expanded in the series of spherical harmonics. In low collisionality regimes effects of electron thermal motion becomes essential so that the higher harmonics are important. This leads to the infinite system of coupled equations for the separate harmonics. We have developed a procedure that allows one to solve this infinite hierarchy in terms of the continued fraction and found a finite parametric representation for this continued fraction. Unlike the two-term approximation, which only accounts for the first term f_1 in the spherical modes expansion, the approach developed in the present paper leads to a space and time dependent electron distribution function that describes electron kinetics when the electron's mean free path is comparable to or exceeds the characteristic length scale of an external electric field inhomogeneity. The developed approach uniformly describes the low and high collisionality regimes and can be applied to a wide class of electron-neutral interaction processes with an angular dependence of the differential cross sections, in particular, for inert gases exhibiting the Ramsauer effect. The electron distribution function obtained in this paper was used to find the nonlocal conductivity and the surface impedance of a semi-infinite plasma and to analyze the anomalous penetration of the electric field into argon plasma. Unlike the local conductivity, which leads to the exponential (local) decay of the electric field, the nonlocal conductivity leads to nonmonotonous dependence of the amplitude of the electric field on the penetration depth (anomalous skin effect).

It is shown that the Ramsauer effect manifests itself in the nonmonotonic behavior of the impedance for small temperatures and becomes noticeable for the gas pressures of the order of 10^{-2} Torr. We have introduced the nonlocality parameter ζ, which determines the boundary between the local, $\zeta \leqslant 1$, and nonlocal, $\zeta > 1$, regimes. The approach developed in this paper can also be extended to analyze the nonlocal effects in the thermal conductivity.

ACKNOWLEDGMENTS

Helpful comments provided by Mr. Mike Elia are acknowledged with gratitude. This research was sponsored by the Natural Sciences and Engineering Research Council of Canada.

APPENDIX A: APPROXIMATE EXPRESSION FOR THE CONTINUED FRACTION

The continued fraction H_1 has the form

$$H_1 = 1 + C_2/1 + C_3/1 + C_4/\cdots. \tag{A1}$$

The C_l coefficients are

$$C_l = \frac{(l^2-1)k^2v^2}{(4l^2-1)(i\omega-\nu_l)(i\omega-\nu_{l-1})}, \tag{A2}$$

where ν_l is given by Eq. (21). For large l coefficients C_l converge to a constant

$$C_l = \frac{1}{4}\,\frac{k^2v^2}{(i\omega-\nu_\infty)^2} = \frac{1}{4}x^2, \tag{A3}$$

where $x = kv/(i\omega-\nu_\infty)$. For constant value of C_l the following identity holds true:

$$\sqrt{1+x^2} = 1 + \frac{1}{2}\,\cfrac{x^2}{1+\frac{1}{4}\,\cfrac{x^2}{1+\frac{1}{4}\,\cfrac{x^2}{1+\frac{1}{4}\,\cfrac{x^2}{1+\ldots}}}}. \tag{A4}$$

Keeping a few first terms exact and replacing the rest with the approximate expression $C_l \simeq x^2/4$ the function H_1 can be represented asymptotically

$$H_1^N = 1 + C_2/1 + C_3/1 + \cdots + C_N/1 + \frac{x^2}{4}\Big/ 1 + \frac{x^2}{4}\Big/ 1 + \cdots. \tag{A5}$$

Then

$$H_1^N = 1 + C_2/1 + C_3/1 + \cdots + C_N/1 + (\sqrt{1+x^2}/2 - 1/2). \tag{A6}$$

Approximating C_3 and higher coefficients by an expression (A3) and using expansion (A4), one finds for H_1 [Eq. (A1)]

$$H_1 \simeq 1 + \cfrac{C_2}{1+\frac{1}{4}\,\cfrac{x^2}{1+\frac{1}{4}\,\cfrac{x^2}{1+\frac{1}{4}\,\cfrac{x^2}{1+\cdots}}}}$$

$$= 1 + \frac{2}{5}\,\frac{k^2v^2}{(i\omega-\nu_2)(i\omega-\nu_1)(\sqrt{1+k^2v^2/(i\omega-\nu_\infty)^2}+1)}. \tag{A7}$$

The accuracy of the expression (A7) can be improved by approximating C_4 and higher coefficients by an expression (A3), whereas C_1, C_2, and C_3 are calculated according to Eq. (A2).

APPENDIX B: THE DIFFERENTIAL CROSS SECTION OF ELECTRON-NEUTRAL ATOM SCATTERING

The differential cross section of the electron neutral atom interaction is given by

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2, \tag{B1}$$

where $f(\theta)$ is the scattering amplitude. The scattering amplitude can be expressed in terms of the phase shifts δ_l [28]:

$$f(\theta) = \sum_{l=0}^{\infty} \frac{1}{2ik}(2l+1)(e^{2i\delta_l}-1)P_l(\cos\theta), \tag{B2}$$

where k is the electron wave number, and $P_l(\cos\theta)$ are Legendre polynomials. If the ground state of the atom does not have a permanent electric quadrupole moment (which is the case for rare-gas atoms), the leading term in the electron-atom interaction potential is of the polarization type [29]. The solution of the Schrodinger equation with the polarization potential $V \sim 1/r^4$ gives the values of the phase shifts (and, consequently, the differential cross section) that describe the experimental data only for low energies (less than 0.1–0.2 eV for argon gas). For higher electron energy, the interaction potential between an electron and an atom deviates from the polarization form, so that the higher order terms in the multipole expansion ($1/r^6$, ...,etc.) of the interaction potential become important. According to the modified effective range theory [29] the phase shifts can be represented as a series in the ka_0 parameter (which is the measure of the energy of the incident electron). Coefficients of this expansion are chosen to fit the experimental data. Following this procedure O'Malley [18] has obtained the following expressions for the phase shifts:

$$\tan\delta_o = -Lk - \frac{\pi\beta k^2}{3a_0} - \frac{4}{3a_0}\beta L k^3 \ln(ka_0) + Dk^3 + O(k^5), \tag{B3}$$

$$\tan\delta_1 = \frac{\pi}{15a_0}\beta k^2 + A_1 k^3 + O(k^4), \tag{B4}$$

$$\tan\delta_l = \pi[(2l+3)(2l+1)(2l-1)a_0]^{-1}\beta k^2 + O(k^4), \quad \text{for } l>1. \tag{B5}$$

where for argon gas

$$L = -1.7a_0, \quad \beta = 11a_0^3, \quad D = 49.368a_0^3, \quad A_1 = -8a_0^3. \tag{B6}$$

The expressions given by Eqs. (B3), (B4), (B5) provide a good fit for the electron-argon atom cross sections in the energy range between 0 and 0.7 eV.

To extend these expressions into the higher energy region we introduce the higher order terms in the modified effective range theory expansion:

$$\tan\delta_0 = -Lk - \frac{\pi\beta k^2}{3a_0} - \frac{4}{3a_0}\beta L k^3 \ln(ka_0) + Dk^3 + \sum_{l=4}^{\alpha} \beta_l k^l, \tag{B7}$$

$$\tan\delta_1 = \frac{\pi}{15a_0}\beta k^2 + A_1 k^3 + \sum_{l=4}^{\mu} \gamma_l k^l, \tag{B8}$$

$$\tan\delta_2 = \frac{\pi}{105}\beta k^2 + \sum_{l=2}^{\delta} j_l k^{2l}, \tag{B9}$$

$$\tan\delta_l = \pi[(2l+3)(2l+1)(2l-1)a_0]^{-1}\beta k^2, \quad l>2. \tag{B10}$$

New coefficients β_l, γ_l, and j_l are chosen to fit the experimentally measured phase shifts [21] to give

$$\beta_4 = -900.1a_0^4, \quad \gamma_4 = -23.287a_0^4, \quad j_2 = 0.77a_0^4,$$

$$\beta_5 = 18121.4a_0^5, \quad \gamma_5 = 354.605a_0^5, \quad j_3 = 0.85a_0^6,$$

$$\beta_6 = -150462.988a_0^6, \quad \gamma_6 = -2386.8a_0^6,$$

$$\beta_7 = 687021.5a_0^7, \quad \gamma_7 = 9514.9a_0^7,$$

$$\beta_8 = -1938761.7a_0^8, \quad \gamma_8 = -23018.38a_0^8,$$

$$\beta_9 = 3533198.63a_0^9, \quad \gamma_9 = 34197.286a_0^9,$$

$$\beta_{10} = -4174116.25a_0^{10}, \quad \gamma_{10} = -305311.53a_0^{10},$$

$$\beta_{11} = 3090292.45a_0^{11}, \quad \gamma_{11} = 15039.7a_0^{11},$$

$$\beta_{12} = -1303452.58a_0^{12}, \quad \gamma_{12} = -3141.58a_0^{12},$$

$$\beta_{13} = 239014.83a_0^{13}.$$

Then, the scattering amplitude $f(\theta)$ and differential cross section $d\sigma/d\Omega$ are

$$f(\theta) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)\delta_l P_l(\cos\theta) = f(\theta) = \hat{A} - \frac{\pi\beta k}{2a_0}\sin\frac{\theta}{2} + \hat{B}P_1(\cos\theta) + \hat{C}P_2(\cos\theta), \tag{B11}$$

$$\begin{aligned}\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = {} & \hat{A}\hat{A}^* - \frac{\pi\beta k}{2a_0}\{\hat{A}+\hat{A}^*\}\sin\frac{\theta}{2} + \{\hat{A}\hat{B}^* + \hat{A}^*\hat{B}\}P_1(\cos\theta) - \frac{\pi\beta k}{2a_0}\{\hat{B}+\hat{B}^*\}\sin\frac{\theta}{2}P_1(\cos\theta) + \frac{\pi^2\beta^2k^2}{4a_0^2}\sin^2\frac{\theta}{2}\\ & + \hat{B}\hat{B}^*P_1^2(\cos\theta) + \hat{C}\hat{C}^*P_2^2(\cos\theta) + \{\hat{A}\hat{C}^* + \hat{A}^*\hat{C}\}P_2(\cos\theta) - \frac{\pi\beta k}{2a_0}\{\hat{C}+\hat{C}^*\}\sin\frac{\theta}{2}P_2(\cos\theta)\\ & + \{\hat{B}\hat{C}^* + \hat{B}^*\hat{C}\}P_1(\cos\theta)P_2(\cos\theta).\end{aligned} \tag{B12}$$

Here $P_1(\cos\theta)$, $P_2(\cos\theta)$ are the first and second order Legendre polynomials,

$$\hat{A}=\frac{e^{i\delta_0}}{k}\sin\delta_0+\frac{\pi\beta k}{3a_0}, \tag{B13}$$

$$\hat{B}=\frac{3e^{i\delta_1}}{k}\sin\delta_1-\frac{\pi\beta k}{5a_0}, \tag{B14}$$

$$\hat{C}=\frac{5e^{i\delta_2}}{k}\sin\delta_2-\frac{\pi\beta k}{21a_0}. \tag{B15}$$

The expansion

$$\sin\frac{\theta}{2}=-2\sum_{l=0}^{\infty}\frac{P_l(\cos\theta)}{(2l+3)(2l-1)} \tag{B16}$$

was used in derivation of formulas (B11) and (B12). The differential cross section (B12) gives a very good description of the electron scattering by the argon atom in the energy range from 0 to 10 eV. This can be extended to even higher energies by including the next order terms in the expansion for the phase shifts.

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